Weyl-Invariant Light-Like Branes and Black Hole Physics*

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Abstract

We propose a new class of p-brane theories which are Weyl-conformally invariant for any p. For any odd world-volume dimension the latter describe intrinsically light-like branes, hence the name *WILL*-branes (Weyl-Invariant Light-Like branes). Next we discuss the dynamics of *WILL*-membranes (i.e., for $p = 2$) both as test branes in various external physically relevant $D = 4$ gravitational backgrounds, as well as within the framework of a coupled $D = 4$ Einstein-Maxwell-*WILL*-membrane system. In all cases we find that the *WILL*-membrane materializes the event horizon of the corresponding black hole solutions, thus providing an explicit dynamical realization of the membrane paradigm in black hole physics.

1 Introduction - Main Motivation

The consistent Lagrangian formulation of geometrically motivated field theories (gravity, strings, branes, *etc.*; for a background on string and brane theories, see refs.[1].) requires among other things reparametrization-covariant (generally-covariant) integration measure densities (volume-forms). The usual choice is the standard Riemannian integration measure given by $\sqrt{-g}$ with $g \equiv \det ||g_{\mu\nu}||$, where $g_{\mu\nu}$ indicates the intrinsic Riemannian metric on the underlying manifold.

However, equally well-suited is the following alternative non-Riemannian integration measure density:

$$\Phi(\varphi) \equiv \frac{1}{D!}\varepsilon^{\mu_1\ldots\mu_D}\varepsilon_{i_1\ldots i_D}\partial_{\mu_1}\varphi^{i_1}\ldots\partial_{\mu_D}\varphi^{i_D} \quad , \quad i = 1,\ldots,D \tag{1}$$

built in terms of D auxiliary scalar fields independent of the intrinsic Riemannian metric.

In a series of papers [2] two of us have proposed new classes of models involving gravity, called *two-measure theories*, whose actions contain both standard Riemannian *and* alternative non-Riemannian integration measures :

$$S = \int d^D x\, \Phi(\varphi)\, L_1 + \int d^D x\, \sqrt{-g}\, L_2$$

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The scalar Lagrangians are of the following generic form:

$$L_1 = e^{\frac{\alpha\phi}{M_P}}\Big[-\frac{1}{\kappa}R(g,\Gamma) - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \big(\text{Higgs}\big) + \big(\text{fermions}\big)\Big]$$

and similarly for L_2 (with different choice of the normalization factors in front of each of the terms). Here $R(g,\Gamma)$ is the scalar curvature in the first order formalism, ϕ is the dilaton field, M_P denotes the Planck mass, *etc.* The auxiliary fields φ^i are pure-gauge degrees of freedom except for the new dynamical "geometric" field $\zeta(x) \equiv \frac{\Phi(\varphi)}{\sqrt{-g}}$, whose dynamics is determined only through the matter fields locally (*i.e.*, without gravitational interaction).

Two-measure theories address various basic problems in cosmology and particle physics, and provide plausible solutions for a broad array of issues, such as:

- Scale invariance and its dynamical breakdown; Spontaneous generation of dimensionfull fundamental scales;
- Cosmological constant problem;
- The problem of fermionic families;
- Applications in modern brane-world scenarios.

For a detailed exposition we refer to the series of papers [2, 3].

Subsequently, the idea of employing an alternative non-Riemannian integration measure was applied systematically to string, p-brane and Dp-brane models [4]. The main feature of these new classes of modified string/brane theories is the appearance of the pertinent string/brane tension as an additional dynamical degree of freedom beyond the usual string/brane physical degrees of freedom, instead of being introduced *ad hoc* as a dimensionfull scale. In the next section we briefly recall the construction of the modified bosonic string model with a dynamical tension before proceeding to our main task. It is the construction of a novel class of p-brane theories which are Weyl-conformal invariant for any p and whose dynamics significantly differs both from the standard Nambu-Goto (or Dirac-Born-Infeld) branes as well as from their modified versions with dynamical string/brane tensions [4] mentioned above.

2 Strings and Branes with a Modified World-Sheet/World-Volume Integration Measure

The modified-measure bosonic string model is given by the following action:

$$S = -\int d^2\sigma\,\Phi(\varphi)\Big[\frac{1}{2}\gamma^{ab}\partial_a X^\mu\partial_b X^\nu G_{\mu\nu}(X) - \frac{\varepsilon^{ab}}{2\sqrt{-\gamma}}F_{ab}(A)\Big] + \int d^2\sigma\,\sqrt{-\gamma}A_a J^a \quad ; \quad J^a = \frac{\varepsilon^{ab}}{\sqrt{-\gamma}}\partial_b u\;, \tag{2}$$

with the notations:

$$\Phi(\varphi) \equiv \frac{1}{2}\varepsilon_{ij}\varepsilon^{ab}\partial_a\varphi^i\partial_b\varphi^j \quad , \quad F_{ab}(A) = \partial_a A_b - \partial_b A_a\;, \tag{3}$$

γ_{ab} denotes the intrinsic Riemannian world-sheet metric with $\gamma = \det\|\gamma_{ab}\|$ and $G_{\mu\nu}(X)$ is the Riemannian metric of the embedding space-time ($a,b = 0,1; i,j = 1,2; \mu,\nu = 0,1,\ldots,D-1$).

Here is the list of differences w.r.t. the standard Nambu-Goto string (in the Polyakov-like formulation) :

- New non-Riemannian integration measure density $\Phi(\varphi)$ instead of $\sqrt{-\gamma}$;
- Dynamical string tension $T \equiv \frac{\Phi(\varphi)}{\sqrt{-\gamma}}$ instead of *ad hoc* dimensionfull constant;
- Auxiliary world-sheet gauge field A_a in a would-be "topological" term $\int d^2\sigma \frac{\Phi(\varphi)}{\sqrt{-\gamma}}\frac{1}{2}\varepsilon^{ab}F_{ab}(A)$;
- Optional natural coupling of auxiliary A_a to external conserved world-sheet electric current J^a (see last equality in (2) and Eq.(5) below).

The modified string model (2) is Weyl-conformally invariant similarly to the ordinary case. Here Weyl-conformal symmetry is given by Weyl rescaling of γ_{ab} supplemented with a special diffeomorphism in φ-target space:

$$\gamma_{ab} \longrightarrow \gamma'_{ab} = \rho\,\gamma_{ab} \quad , \quad \varphi^i \longrightarrow \varphi'^{\,i} = \varphi'^{\,i}(\varphi) \;\; \text{with} \;\; \det\Bigl\|\frac{\partial\varphi'^{\,i}}{\partial\varphi^j}\Bigr\| = \rho \,. \tag{4}$$

The dynamical string tension appears as a canonically conjugated momentum w.r.t. A_1: $\pi_{A_1} \equiv \frac{\partial\mathcal{L}}{\partial\dot{A}_1} = \frac{\Phi(\varphi)}{\sqrt{-\gamma}} \equiv T$, *i.e.*, T has the meaning of a *world-sheet electric field strength*, and the eqs. of motion w.r.t. auxiliary gauge field A_a look exactly as $D=2$ Maxwell eqs.:

$$\frac{\varepsilon^{ab}}{\sqrt{-\gamma}}\partial_b T + J^a = 0 \,. \tag{5}$$

In particular, for $J^a = 0$:

$$\varepsilon^{ab}\partial_b\Bigl(\frac{\Phi(\varphi)}{\sqrt{-\gamma}}\Bigr) = 0 \qquad , \quad \frac{\Phi(\varphi)}{\sqrt{-\gamma}} \equiv T = \text{const} \,, \tag{6}$$

one gets a *spontaneously induced* constant string tension. Furthermore, when the modified string couples to point-like charges on the world-sheet (*i.e.*, $J^0\sqrt{-\gamma} = \sum_i e_i\delta(\sigma-\sigma_i)$ in (5)) one obtains classical charge *confinement*: $\sum_i e_i = 0$.

The above charge confinement mechanism has also been generalized in [4] to the case of coupling the modified string model with dynamical tension to non-Abelian world-sheet "color" charges. The latter is achieved as follows. Notice the following identity in $2D$ involving Abelian gauge field A_a:

$$\frac{\varepsilon^{ab}}{2\sqrt{-\gamma}}F_{ab}(A) = \sqrt{-\frac{1}{2}F_{ab}(A)F_{cd}(A)\gamma^{ac}\gamma^{bd}} \,. \tag{7}$$

Then the extension of the action (2) to the non-Abelian case is straightforward:

$$S = -\int d^2\sigma\,\Phi(\varphi)\Bigl[\frac{1}{2}\gamma^{ab}\partial_a X^\mu\partial_b X^\nu G_{\mu\nu}(X) - \sqrt{-\frac{1}{2}\,\mathrm{Tr}(F_{ab}(A)F_{cd}(A))\gamma^{ac}\gamma^{bd}}\Bigr] + \int d^2\sigma\;\mathrm{Tr}\,(A_a j^a) \tag{8}$$

with $F_{ab}(A) = \partial_a A_b - \partial_b A_c + i\bigl[A_a,\, A_b\bigr]$, sharing the same principal property – dynamical generation of string tension as an additional degree of freedom.

3 New Class of Weyl-Invariant p-Brane Theories

3.1 Weyl-Invariant Branes: Action and Equations of Motion

The identity (7) suggests how to construct **Weyl-invariant** p-brane models for any p. Namely, we propose the following novel p-brane actions:

$$S = -\int d^{p+1}\sigma\,\Phi(\varphi)\Bigl[\frac{1}{2}\gamma^{ab}\partial_a X^\mu\partial_b X^\nu G_{\mu\nu}(X) - \sqrt{F_{ab}(A)F_{cd}(A)\gamma^{ac}\gamma^{bd}}\Bigr] \tag{9}$$

$$\Phi(\varphi) \equiv \frac{1}{(p+1)!}\varepsilon_{i_1\ldots i_{p+1}}\varepsilon^{a_1\ldots a_{p+1}}\partial_{a_1}\varphi^{i_1}\ldots\partial_{a_{p+1}}\varphi^{i_{p+1}} \; , \tag{10}$$

where notations similar to those in (2) are used (here $a, b = 0, 1, \ldots, p; i, j = 1, \ldots, p+1$).

The above action (9) is invariant under Weyl-conformal symmetry (the same as in the dynamical-tension string model (2)):

$$\gamma_{ab} \longrightarrow \gamma'_{ab} = \rho\,\gamma_{ab} \quad , \quad \varphi^i \longrightarrow \varphi'^{\,i} = \varphi'^{\,i}(\varphi) \;\; \text{with} \;\; \det\Bigl\|\frac{\partial\varphi'^{\,i}}{\partial\varphi^j}\Bigr\| = \rho \; . \tag{11}$$

We notice the following significant differences of (9) w.r.t. the standard Nambu-Goto p-branes (in the Polyakov-like formulation) :

- New non-Riemannian integration measure density $\Phi(\varphi)$ instead of $\sqrt{-\gamma}$, and *no* "cosmological-constant" term $((p-1)\sqrt{-\gamma})$;
- Variable brane tension $\chi \equiv \frac{\Phi(\varphi)}{\sqrt{-\gamma}}$ which is Weyl-conformal *gauge dependent*: $\chi \to \rho^{\frac{1}{2}(1-p)}\chi$;
- Auxiliary world-volume gauge field A_a in a "square-root" Maxwell (Yang-Mills) term[1]; the latter is straightforwardly generalized to the non-Abelian case – $\sqrt{-\,\mathrm{Tr}\,(F_{ab}(A)F_{cd}(A))\,\gamma^{ac}\gamma^{bd}}$ similarly to (8);
- Natural optional couplings of the auxiliary gauge field A_a to external world-volume "color" charge currents j^a;
- The action (9) is manifestly Weyl-conformal invariant for *any* p; it describes *intrinsically light-like* p-branes for any even p.

The eqs. of motion w.r.t. measure-building auxiliary scalars φ^i are:

$$\frac{1}{2}\gamma^{cd}\left(\partial_c X \partial_d X\right) - \sqrt{FF\gamma\gamma} = M \,\Bigl(= \text{const}\Bigr) \; , \tag{12}$$

employing the short-hand notations:

$$\left(\partial_a X \partial_b X\right) \equiv \partial_a X^\mu \partial_b X^\nu G_{\mu\nu} \quad , \quad \sqrt{FF\gamma\gamma} \equiv \sqrt{F_{ab}F_{cd}\gamma^{ac}\gamma^{bd}} \; . \tag{13}$$

The eqs. of motion w.r.t. γ^{ab} read:

$$\frac{1}{2}\left(\partial_a X \partial_b X\right) + \frac{F_{ac}\gamma^{cd}F_{db}}{\sqrt{FF\gamma\gamma}} = 0 \; , \tag{14}$$

and (upon taking the trace) imply $M = 0$ in Eq.(12).

Next we have the following eqs. of motion w.r.t. auxiliary gauge field A_a and w.r.t. X^μ, respectively:

$$\partial_b\left(\frac{F_{cd}\gamma^{ac}\gamma^{bd}}{\sqrt{FF\gamma\gamma}}\Phi(\varphi)\right) = 0 \; , \tag{15}$$

$$\partial_a\Bigl(\Phi(\varphi)\gamma^{ab}\partial_b X^\mu\Bigr) + \Phi(\varphi)\gamma^{ab}\partial_a X^\nu \partial_b X^\lambda \Gamma^\mu_{\nu\lambda} = 0 \; , \tag{16}$$

where $\Gamma^\mu_{\nu\lambda} = \frac{1}{2}G^{\mu\kappa}\left(\partial_\nu G_{\kappa\lambda} + \partial_\lambda G_{\kappa\nu} - \partial_\kappa G_{\nu\lambda}\right)$ is the affine connection corresponding to the external space-time metric $G_{\mu\nu}$.

[1] "Square-root" Maxwell (Yang-Mills) action in $D = 4$ was originally introduced in the first ref.[5] and later generalized to "square-root" actions of higher-rank antisymmetric tensor gauge fields in $D \geq 4$ in the second and third refs.[5].

3.2 Intrinsically Light-Like Branes

Let us consider the γ^{ab}-eqs. of motion (14). F_{ab} is an anti-symmetric $(p+1) \times (p+1)$ matrix, therefore, F_{ab} is *not invertible* in any odd $(p+1)$ – it has at least one zero-eigenvalue vector V^a ($F_{ab}V^b = 0$). Therefore, for any odd $(p+1)$ the induced metric:

$$g_{ab} \equiv (\partial_a X \partial_b X) \equiv \partial_a X^\mu \partial_b X^\nu G_{\mu\nu} \tag{17}$$

on the world-volume of the Weyl-invariant brane (9) is *singular* as *opposed* to the ordinary Nambu-Goto brane (where the induced metric is proportional to the intrinsic Riemannian world-volume metric):

$$(\partial_a X \partial_b X)\, V^b = 0 \quad , \quad \text{i.e.} \;\; (\partial_V X \partial_V X) = 0 \;\; , \;\; (\partial_\perp X \partial_V X) = 0 \; , \tag{18}$$

where $\partial_V \equiv V^a \partial_a$ and $\partial_\perp$ are derivates along the tangent vectors in the complement of the tangent vector field V^a.

Thus, we arrive at the following important conclusion: every point on the world-surface of the Weyl-invariant p-brane (9) (for odd $(p+1)$) moves with the speed of light in a time-evolution along the zero-eigenvalue vector-field V^a of the world-volume electromagnetic field-strength F_{ab}. Therefore, we will name (9) (for odd $(p+1)$) by the acronym *WILL-brane* (Weyl-Invariant Light-Like-brane) model.

3.3 Dual Formulation of *WILL*-Branes

The A_a-eqs. of motion (15) can be solved in terms of $(p-2)$-form gauge potentials $\Lambda_{a_1 \ldots a_{p-2}}$ dual w.r.t. A_a. The respective field-strengths are related as follows:

$$F_{ab}(A) = -\frac{1}{\chi} \frac{\sqrt{-\gamma}\, \varepsilon_{abc_1 \ldots c_{p-1}}}{2(p-1)} \gamma^{c_1 d_1} \ldots \gamma^{c_{p-1} d_{p-1}}\, F_{d_1 \ldots d_{p-1}}(\Lambda)\, \gamma^{cd} \left(\partial_c X \partial_d X\right) \; , \tag{19}$$

$$\chi^2 = -\frac{2}{(p-1)^2}\, \gamma^{a_1 b_1} \ldots \gamma^{a_{p-1} b_{p-1}} F_{a_1 \ldots a_{p-1}}(\Lambda) F_{b_1 \ldots b_{p-1}}(\Lambda) \; , \tag{20}$$

where $\chi \equiv \frac{\Phi(\varphi)}{\sqrt{-\gamma}}$ is the variable brane tension, and:

$$F_{a_1 \ldots a_{p-1}}(\Lambda) = (p-1) \partial_{[a_1} \Lambda_{a_2 \ldots a_{p-1}]} \tag{21}$$

is the $(p-1)$-form dual field-strength.

All eqs. of motion can be equivalently derived from the following *dual WILL*-brane action:

$$S_{\rm dual} = -\frac{1}{2} \int d^{p+1}\sigma\, \chi(\gamma, \Lambda) \sqrt{-\gamma} \gamma^{ab} \partial_a X^\mu \partial_b X^\nu G_{\mu\nu} \tag{22}$$

with $\chi(\gamma, \Lambda)$ given in (20) above.

4 Special case $p = 2$: *WILL*-Membrane

The *WILL*-membrane dual action (particular case of (22) for $p = 2$) reads:

$$S_{\rm dual} = -\frac{1}{2} \int d^3\sigma\, \chi(\gamma, u)\, \sqrt{-\gamma} \gamma^{ab} \left(\partial_a X \partial_b X\right) \; , \tag{23}$$

$$\chi(\gamma, u) \equiv \sqrt{-2\gamma^{cd} \partial_c u \partial_d u} \; , \tag{24}$$

where u is the dual "gauge" potential w.r.t. A_a:

$$F_{ab}(A) = -\frac{1}{2\chi(\gamma,u)}\sqrt{-\gamma}\varepsilon_{abc}\gamma^{cd}\partial_d u\,\gamma^{ef}(\partial_e X\partial_f X) \ . \tag{25}$$

$S_{\rm dual}$ is manifestly Weyl-invariant (under $\gamma_{ab} \to \rho\gamma_{ab}$).

The eqs. of motion w.r.t. γ^{ab}, u (or A_a), and X^μ read accordingly:

$$(\partial_a X\partial_b X) + \frac{1}{2}\gamma^{cd}(\partial_c X\partial_d X)\left(\frac{\partial_a u\partial_b u}{\gamma^{ef}\partial_e u\partial_f u} - \gamma_{ab}\right) = 0 \ , \tag{26}$$

$$\partial_a\left(\frac{\sqrt{-\gamma}\gamma^{ab}\partial_b u}{\chi(\gamma,u)}\gamma^{cd}(\partial_c X\partial_d X)\right) = 0 \ , \tag{27}$$

$$\partial_a\left(\chi(\gamma,u)\sqrt{-\gamma}\gamma^{ab}\partial_b X^\mu\right) + \chi(\gamma,u)\sqrt{-\gamma}\gamma^{ab}\partial_a X^\nu\partial_b X^\lambda\Gamma^\mu_{\nu\lambda} = 0 \ . \tag{28}$$

The first eq. above shows that the induced metric $g_{ab} \equiv (\partial_a X\partial_b X)$ has zero-mode eigenvector $V^a = \gamma^{ab}\partial_b u$.

The invariance under world-volume reparametrizations allows to introduce the following standard (synchronous) gauge-fixing conditions:

$$\gamma^{0i} = 0 \ \ (i=1,2) \quad , \quad \gamma^{00} = -1 \ . \tag{29}$$

In what follows we will use the ansatz for the dual "gauge potential":

$$u(\tau,\sigma^1,\sigma^2) = \frac{T_0}{\sqrt{2}}\tau \ , \tag{30}$$

where T_0 is an arbitrary integration constant with the dimension of membrane tension. In particular:

$$\chi \equiv \sqrt{-2\gamma^{ab}\partial_a u\partial_b u} = T_0 \tag{31}$$

This means that we take $\tau \equiv \sigma^0$ to be evolution parameter along the zero-eigenvalue vector-field of the induced metric on the brane ($V^a = \gamma^{ab}\partial_b u = \text{const}\,(1,0,0)$).

The ansatz for u (30) together with the gauge choice for γ_{ab} (29) brings the eqs. of motion w.r.t. γ^{ab}, u (or A_a) and X^μ in the following form (recall $(\partial_a X\partial_b X) \equiv \partial_a X^\mu\partial_b X^\nu G_{\mu\nu}$):

$$(\partial_0 X\partial_0 X) = 0 \quad , \quad (\partial_0 X\partial_i X) = 0 \ , \tag{32}$$

$$(\partial_i X\partial_j X) - \frac{1}{2}\gamma_{ij}\gamma^{kl}(\partial_k X\partial_l X) = 0 \ , \tag{33}$$

(Eqs.(33) look exactly like the classical (Virasoro) constraints for an Euclidean string theory with world-sheet parameters (σ^1,σ^2));

$$\partial_0\left(\sqrt{\gamma_{(2)}}\gamma^{kl}(\partial_k X\partial_l X)\right) = 0 \ , \tag{34}$$

where $\gamma_{(2)} = \det\|\gamma_{ij}\|$ (the above equation is the only remnant from the A_a-eqs. of motion (15));

$$\Box^{(3)}X^\mu + \left(-\partial_0 X^\nu\partial_0 X^\lambda + \gamma^{kl}\partial_k X^\nu\partial_l X^\lambda\right)\Gamma^\mu_{\nu\lambda} = 0 \ , \tag{35}$$

where:

$$\Box^{(3)} \equiv -\frac{1}{\sqrt{\gamma^{(2)}}}\partial_0\left(\sqrt{\gamma^{(2)}}\partial_0\right) + \frac{1}{\sqrt{\gamma^{(2)}}}\partial_i\left(\sqrt{\gamma^{(2)}}\gamma^{ij}\partial_j\right) \ . \tag{36}$$

5 *WILL*-Membrane Solutions in Various Gravitational Backgrounds

5.1 Example: WILL-Membrane in a PP-Wave Background

As a simplest non-trivial example let us consider in (23) external space-time metric $G_{\mu\nu}$ for plane-polarized gravitational wave (*pp-wave*) background:

$$(ds)^2 = -dx^+ dx^- - F(x^+, x^I)\,(dx^+)^2 + dx^I dx^I \ , \tag{37}$$

and employ in (32)–(36) the following natural ansatz for X^μ (here $\sigma^0 \equiv \tau$; $I = 1, \ldots, D-2$) :

$$X^- = \tau \quad , \quad X^+ = X^+(\tau, \sigma^1, \sigma^2) \quad , \quad X^I = X^I(\sigma^1, \sigma^2) \ . \tag{38}$$

The non-zero affine connection symbols for the pp-wave metric (37) are: $\Gamma^-_{++} = \partial_+ F$, $\Gamma^-_{+I} = \partial_I F$, $\Gamma^I_{++} = \frac{1}{2}\partial_I F$.

It is straightforward to show that the solution does not depend on the form of the pp-wave front $F(x^+, x^I)$ and reads:

$$X^+ = X^+_0 = \text{const} \quad , \quad \gamma_{ij} = \tau\!-\!\text{independent} \ ; \tag{39}$$

$$\partial_i X^I \partial_j X^I - \frac{1}{2}\gamma_{ij}\gamma^{kl}\partial_k X^I \partial_l X^I = 0 \quad , \quad \partial_i \left(\sqrt{\gamma^{(2)}}\gamma^{ij}\partial_j X^I\right) = 0 \tag{40}$$

where the latter eqs. describe a string embedded in the transverse $(D-2)$-dimensional flat Euclidean space.

5.2 Example: WILL-Membrane in a Schwarzschild Black Hole

Let us consider spherically-symmetric static gravitational background:

$$(ds)^2 = -A(r)(dt)^2 + B(r)(dr)^2 + r^2[(d\theta)^2 + \sin^2(\theta)\,(d\phi)^2] \ . \tag{41}$$

For the Schwarzschild black hole we have $A(r) = B^{-1}(r) = 1 - \frac{2GM}{r}$.

We find the following solution to the eqs. of motion (and constraints) (32)–(36). Using the ansatz:

$$X^0 \equiv t = \tau \quad , \quad X^1 \equiv r = r(\tau, \sigma^1, \sigma^2) \quad , \quad X^2 \equiv \theta = \theta(\sigma^1, \sigma^2) \quad , \quad X^3 \equiv \phi = \phi(\sigma^1, \sigma^2) \ , \tag{42}$$

$$\gamma_{ij} = a(\tau)\,\widetilde{\gamma}_{ij}(\sigma^1, \sigma^2) \ , \tag{43}$$

with $\widetilde{\gamma}_{ij}$ being some standard reference $2D$ metric on the membrane surface ($i,j = 1,2$), we obtain from Eqs.(32) taking into account (41):

$$\frac{\partial}{\partial\tau} r = \pm A(r) \quad , \quad \frac{\partial}{\partial\sigma^i} r = 0 \ . \tag{44}$$

From Eq.(34) we get $\frac{\partial}{\partial\tau} r = 0$ which upon combining with (44) gives:

$$r = r_0 \equiv 2GM = \text{const} \quad , \quad i.e.\ \ A(r_0) = 0 \ . \tag{45}$$

For the rest of embedding coordinates and the intrinsic *WILL*-membrane metric (upon assuming the membrane surface to be of spherical topology) we obtain:

$$\theta = \sigma^1 \quad , \quad \phi = \sigma^2 \quad , \quad \|\gamma_{ij}\| = c_0\, e^{\mp\tau/r_0} \begin{pmatrix} 1 & 0 \\ 0 & \sin^2(\sigma^1) \end{pmatrix} \ , \tag{46}$$

where c_0 is an arbitrary integration constant.

That is, the *WILL*-membrane with spherical topology (and with exponentially blowing-up/deflating internal metric) “sits” on (materializes) the event horizon of the Schwarzschild black hole.

5.3 Example: WILL-Membrane in a Reissner-Nordström Black Hole

Now we need to extend the *WILL*-brane model (9) via a coupling to external space-time electromagnetic field $\mathcal{A}_\mu$. The natural Weyl-conformal invariant candidate action reads (for $p = 2$):

$$S = -\int d^3\sigma\,\Phi(\varphi)\Big[\frac{1}{2}\gamma^{ab}\partial_a X^\mu \partial_b X^\nu G_{\mu\nu} - \sqrt{F_{ab}F_{cd}\gamma^{ac}\gamma^{bd}}\Big] - q\int d^3\sigma\,\varepsilon^{abc}\mathcal{A}_\mu \partial_a X^\mu F_{bc}\,. \tag{47}$$

The last Chern-Simmons-like term is a special case of a class of Chern-Simmons-like couplings of extended objects to external electromagnetic fields proposed in ref.[6].

In the dual formulation we get accordingly:

$$S_{\rm dual} = -\frac{1}{2}\int d^3\sigma\,\chi(\gamma,u,\mathcal{A})\sqrt{-\gamma}\gamma^{ab}\left(\partial_a X\partial_b X\right)\;, \tag{48}$$

with a variable brane tension:

$$\chi(\gamma,u,\mathcal{A}) \equiv \sqrt{-2\gamma^{cd}\left(\partial_c u - q\mathcal{A}_c\right)\left(\partial_d u - q\mathcal{A}_d\right)} \quad , \quad \mathcal{A}_a \equiv \mathcal{A}_\mu \partial_a X^\mu\;. \tag{49}$$

Here u is the dual "gauge" potential w.r.t. A_a and the corresponding field-strength and dual field-strength are related as:

$$F_{ab}(A) = -\frac{1}{2\chi(\gamma,u,\mathcal{A})}\sqrt{-\gamma}\varepsilon_{abc}\gamma^{cd}\left(\partial_d u - q\mathcal{A}_d\right)\gamma^{ef}\left(\partial_e X\partial_f X\right)\;. \tag{50}$$

The extended *WILL*-membrane model in the dual formulation (48) is likewise manifestly Weyl-invariant (under $\gamma_{ab} \to \rho\gamma_{ab}$).

The eqs. of motion w.r.t. γ^{ab}, u (or A_a), and X^μ read accordingly:

$$\left(\partial_a X\partial_b X\right) + \frac{1}{2}\gamma^{cd}\left(\partial_c X\partial_d X\right)\left(\frac{\left(\partial_a u - q\mathcal{A}_a\right)\left(\partial_b u - q\mathcal{A}_b\right)}{\gamma^{ef}\left(\partial_e u - q\mathcal{A}_e\right)\left(\partial_f u - q\mathcal{A}_f\right)} - \gamma_{ab}\right) = 0\;; \tag{51}$$

$$\partial_a\left(\frac{\sqrt{-\gamma}\gamma^{ab}\left(\partial_b u - q\mathcal{A}_b\right)}{\chi(\gamma,u,\mathcal{A})}\gamma^{cd}\left(\partial_c X\partial_d X\right)\right) = 0\;; \tag{52}$$

$$\begin{aligned}\partial_a\Big(\chi(\gamma,u,\mathcal{A})\sqrt{-\gamma}\gamma^{ab}\partial_b X^\mu\Big) + \chi(\gamma,u,\mathcal{A})\sqrt{-\gamma}\gamma^{ab}\partial_a X^\nu \partial_b X^\lambda \Gamma^\mu_{\nu\lambda}& \\ - q\varepsilon^{abc}F_{bc}\partial_a X^\nu\left(\partial_\lambda \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\lambda\right)G^{\lambda\mu} = 0\;.&\end{aligned} \tag{53}$$

Using the same (synchronous) gauge choice (29) and ansatz for the dual "gauge potential" (30), as well as considering static external space-time electric field ($\mathcal{A}_0 = Q/\sqrt{4\pi}\,r$ – relevant case for Reissner-Nordström blackholes, see next Section), the eqs. of motion w.r.t. γ^{ab}, u (or A_a) and X^μ acquire the form (recall $(\partial_a X\partial_b X) \equiv \partial_a X^\mu \partial_b X^\nu G_{\mu\nu}$):

$$\left(\partial_0 X\partial_0 X\right) = 0 \quad , \quad \left(\partial_0 X\partial_i X\right) = 0\;, \tag{54}$$

$$\left(\partial_i X\partial_j X\right) - \frac{1}{2}\gamma_{ij}\gamma^{kl}\left(\partial_k X\partial_l X\right) = 0\;, \tag{55}$$

(these constraints are the same as in the absence of coupling to space-time gauge field (32)–(33));

$$\partial_0\left(\sqrt{\gamma_{(2)}}\gamma^{kl}\left(\partial_k X\partial_l X\right)\right) = 0\;, \tag{56}$$

(once again the same equation as in the absence of coupling to space-time gauge field (34));

$$\widetilde{\Box}^{(3)} X^\mu + \left(-\partial_0 X^\nu \partial_0 X^\lambda + \gamma^{kl} \partial_k X^\nu \partial_l X^\lambda\right) \Gamma^\mu_{\nu\lambda} - q \frac{\gamma^{kl} \left(\partial_k X \partial_l X\right)}{\sqrt{2}\,\chi} \partial_0 X^\nu \left(\partial_\lambda \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\lambda\right) G^{\lambda\mu} = 0 \,, \tag{57}$$

where $\chi \equiv T_0 - \sqrt{2} q \mathcal{A}_0$ (the variable brane tension), $\mathcal{A}_1 = \ldots = \mathcal{A}_{D-1} = 0$, and:

$$\widetilde{\Box}^{(3)} \equiv -\frac{1}{\chi\sqrt{\gamma^{(2)}}} \partial_0 \left(\chi\sqrt{\gamma^{(2)}} \partial_0\right) + \frac{1}{\chi\sqrt{\gamma^{(2)}}} \partial_i \left(\chi\sqrt{\gamma^{(2)}} \gamma^{ij} \partial_j\right) \,. \tag{58}$$

Now, let us solve Eqs.(54)–(58) in Reissner-Nordström background:

$$(ds)^2 = -A(r)(dt)^2 + A^{-1}(dr)^2 + r^2[(d\theta)^2 + \sin^2(\theta)\,(d\phi)^2] \tag{59}$$

$$A(r) = 1 - \frac{2GM}{r} + \frac{GQ^2}{r^2}. \tag{60}$$

Employing the same ansatz (42) as in the case of Schwarzschild background, the solution for Reissner-Nordström background reads:

$$X^0 \equiv t = \tau \quad , \quad \theta = \sigma^1 \quad , \quad \phi = \sigma^2 \tag{61}$$

$$r(\tau, \sigma^1, \sigma^2) = r_{\text{horizon}} = \text{const} \tag{62}$$

where $A(r_{\text{horizon}}) = 0$;

$$\|\gamma_{ij}\| = \left(c_0\, e^{\mp\tau \left(\frac{\partial}{\partial r} A\right)_{r=r_{\text{horizon}}}} + \frac{qQ}{\sqrt{2\pi}\left(\chi \frac{\partial}{\partial r} A\right)_{r=r_{\text{horizon}}}} \right) \begin{pmatrix} 1 & 0 \\ 0 & \sin^2(\sigma^1) \end{pmatrix} \tag{63}$$

where c_0 is an arbitrary integration constant (recall $\chi \equiv T_0 - \sqrt{2} q \mathcal{A}_0$).

In particular, taking $c_0 = 0$ one obtains the usual time-independent internal spherical metric on the brane surface. Thus, similar to the Schwarzschild case, the *WILL*-membrane with spherical topology "sits" on (materializes) the event horizon of the Reissner-Nordström black hole.

6 Coupled Einstein-Maxwell-*WILL*-Membrane System

We can extend the results from the previous section to the case of the full coupled Einstein-Maxwell-*WILL*-membrane system, *i.e.*, taking into account the back-reaction of the *WILL*-membrane serving as a material and electrically charged source for gravity and electromagnetism. The pertinent action reads:

$$S = \int d^4x \sqrt{-G} \left[\frac{R}{16\pi G_N} - \frac{1}{4} \mathcal{F}_{\mu\nu}(\mathcal{A}) \mathcal{F}_{\kappa\lambda}(\mathcal{A}) G^{\mu\kappa} G^{\nu\lambda}\right] + S_{\text{WILL-brane}} \,, \tag{64}$$

where $\mathcal{F}_{\mu\nu}(\mathcal{A}) = \partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu$, and:

$$S_{\text{WILL-brane}} = -\int d^3\sigma\, \Phi(\varphi) \left[\frac{1}{2} \gamma^{ab} \partial_a X^\mu \partial_b X^\nu G_{\mu\nu} - \sqrt{F_{ab} F_{cd} \gamma^{ac} \gamma^{bd}}\right] - q \int d^3\sigma\, \varepsilon^{abc} \mathcal{A}_\mu \partial_a X^\mu F_{bc} \,. \tag{65}$$

Eqs. of motion for the *WILL*-membrane subsystem are the same as above, namely Eqs.(54)–(58). The rest of the eqs. of motion are:

$$R_{\mu\nu} - \frac{1}{2} G_{\mu\nu} R = 8\pi G_N \left(T^{(EM)}_{\mu\nu} + T^{(brane)}_{\mu\nu}\right) \,, \tag{66}$$

$$\partial_\nu \left(\sqrt{-G}G^{\mu\kappa}G^{\nu\lambda}\mathcal{F}_{\kappa\lambda}\right) + j^\mu = 0 \; , \tag{67}$$

where:

$$T^{(EM)}_{\mu\nu} \equiv \mathcal{F}_{\mu\kappa}\mathcal{F}_{\nu\lambda}G^{\kappa\lambda} - G_{\mu\nu}\frac{1}{4}\mathcal{F}_{\rho\kappa}\mathcal{F}_{\sigma\lambda}G^{\rho\sigma}G^{\kappa\lambda} \; , \tag{68}$$

$$T^{(brane)}_{\mu\nu} \equiv -G_{\mu\kappa}G_{\nu\lambda}\int d^3\sigma \, \frac{\delta^{(4)}\Big(x - X(\sigma)\Big)}{\sqrt{-G}} \, \chi\sqrt{-\gamma}\gamma^{ab}\partial_a X^\kappa \partial_b X^\lambda \; , \tag{69}$$

(recall $\chi \equiv \sqrt{-2\gamma^{cd}\left(\partial_c u - q\mathcal{A}_c\right)\left(\partial_d u - q\mathcal{A}_d\right)}$, $\mathcal{A}_a \equiv \mathcal{A}_\mu \partial_a X^\mu$),

$$j^\mu \equiv q\int d^3\sigma \, \delta^{(4)}\Big(x - X(\sigma)\Big)\varepsilon^{abc}F_{bc}\partial_a X^\mu \; . \tag{70}$$

Following the same steps as in the previous section we obtain the following spherically symmetric stationary solution. For the Einstein subsystem we find a solution:

$$(ds)^2 = -A(r)(dt)^2 + A^{-1}(dr)^2 + r^2[(d\theta)^2 + \sin^2(\theta)\,(d\phi)^2] \; , \tag{71}$$

consisting of two different black holes with a *common* event horizon:

- Schwarzschild black hole inside the horizon:

$$A(r) \equiv A_-(r) = 1 - \frac{2GM_1}{r} \quad , \quad \text{for} \;\; r < r_0 \equiv r_{\text{horizon}} = 2GM_1 \; . \tag{72}$$

- Reissner-Norström black hole outside the horizon:

$$A(r) \equiv A_+(r) = 1 - \frac{2GM_2}{r} + \frac{GQ^2}{r^2} \quad , \quad \text{for} \;\; r > r_0 \equiv r_{\text{horizon}} \; , \tag{73}$$

  where $Q^2 = 8\pi q^2 r^4_{\text{horizon}} \equiv 128\pi q^2 G^4 M_1^4$;

For the Maxwell subsystem we get $\mathcal{A}_1 = \ldots = \mathcal{A}_{D-1} = 0$ everywhere and:

- Coulomb field outside horizon:

$$\mathcal{A}_0 = \frac{\sqrt{2}\,q\,r^2_{\text{horizon}}}{r} \quad , \quad \text{for} \;\; r \geq r_0 \equiv r_{\text{horizon}} \; . \tag{74}$$

- No electric field inside horizon:

$$\mathcal{A}_0 = \sqrt{2}\,q\,r_{\text{horizon}} = \text{const} \quad , \quad \text{for} \;\; r \leq r_0 \equiv r_{\text{horizon}} \; . \tag{75}$$

The *WILL*-membrane again "sits" on (materializes) the common event horizon of the pertinent black holes:

$$X^0 \equiv t = \tau \quad , \quad \theta = \sigma^1 \quad , \quad \phi = \sigma^2 \quad , \quad r(\tau, \sigma^1, \sigma^2) = r_{\text{horizon}} = \text{const} \tag{76}$$

In addition there is an important matching condition for the metric components along the *WILL*-membrane:

$$\frac{\partial}{\partial r}A_+\Big|_{r=r_{\text{horizon}}} - \frac{\partial}{\partial r}A_-\Big|_{r=r_{\text{horizon}}} = -16\pi G\chi \; , \tag{77}$$

which yields the following relations between the parameters of the black holes and the *WILL*-membrane (q being its surface charge density) :

$$M_2 = M_1 + 32\pi q^2 G^3 M_1^3 \qquad (78)$$

and for the brane tension χ:

$$\chi \equiv T_0 - 2q^2 r_{\rm horizon} = q^2 G M_1 \quad , \text{ i.e. } T_0 = 5q^2 G M_1 \qquad (79)$$

The matching condition (77) corresponds to the statically soldering conditions in the light-like thin shell dynamics in general relativity [7]. On the other hand we should stress that unlike the latter phenomenological models of thin shell dynamics (*i.e.*, where the membranes are introduced *ad hoc*), the present *WILL*-brane models provide a systematic description of light-like branes from first principles starting with concise Weyl-conformal invariant actions (9), (64)–(65). As a consequence, these actions also yield additional information impossible to obtain within the phenomenological approach, such as the requirement that the light-like brane must sit on the event horizon of the pertinent black hole.

7 Conclusions and Outlook

In the present work we have demonstrated that employing alternative non-Riemannian world-sheet/world-volume integration measure significantly affects string and p-brane dynamics:

- Acceptable dynamics in the novel class of string/brane models (Eqs.(2) and (9)) *naturally* requires the introduction of auxiliary world-sheet/world-volume gauge fields.
- By employing square-root Yang-Mills actions for the auxiliary world-sheet/world-volume gauge fields one achieves manifest *Weyl-conformal symmetry* in the new class of p-brane theories *for any p*.
- The string/brane tension is *not* a constant dimensionful scale given *ad hoc*, but rather it appears as an *additional dynamical degree of freedom* beyond the ordinary string/brane degrees of freedom.
- The novel class of Weyl-invariant p-brane theories describes intrinsically *light-like* p-branes for any even p (*WILL*-branes).
- When put in a gravitational black hole background, the *WILL*-membrane ($p = 2$) sits on ("materializes") the event horizon.
- The coupled Einstein-Maxwell-*WILL*-membrane system (64) possesses self-consistent solution where the *WILL*-membrane serves as a material and electrically charged source for gravity and electromagnetism, and it "sits" on (materializes) the common event horizon for a Schwarzschild (in the interior) and Reissner-Nordström (in the exterior) black holes. Thus our model (64) provides an explicit dynamical realization of the so called "membrane paradigm" in the physics of black holes [8].

One can think of various physically interesting directions of further reseach on the novel class of Weyl-conformal invariant p-branes such as: quantization (Weyl-conformal anomaly and critical dimensions); supersymmetric extension; possible relevance for the open string dynamics (similar to

the Dirichlet- (Dp-)branes); *WILL*-brane dynamics in more complicated gravitational black hole backgrounds (*e.g.*, Kerr-Newman) *etc.*.

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